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the lectures pdfs are available at:

https://www.physics.umd.edu/rgroups/amo/orozco/results/2022/Results22.htm

Correlations in Optics and Quantum Optics; A series of lectures about correlations and coherence. November 2022

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NIST

Lesson 9

Tentative list of topics to cover:

- From statistics and linear algebra to power spectral densities
- Historical perspectives and examples in many areas of physics
- Correlation functions in classical optics (field-field; intensity-intensity; field-intensity) part iii
- Optical Cavity QED
- Correlation functions, quantum examples
- Correlations and conditional dynamics for control
- Correlations of the field and intensity
- From Cavity QED to waveguide QED.

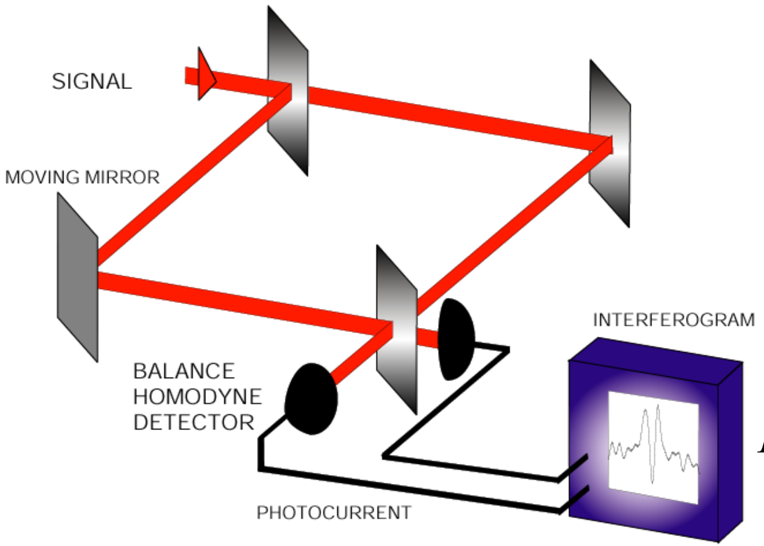
Correlation functions tell us something about the fluctuations.

Correlations have classical bounds.

They are conditional measurements.

Can we use them to measure the field associated with a FLUCTUATION of one photon?

Mach Zehnder Interferometer Wave-Wave Correlation



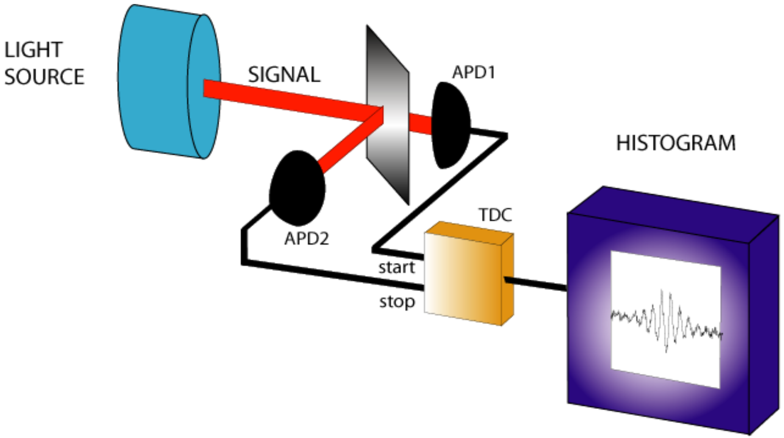
$$g^{(1)}(\tau) = \frac{\langle E^*(t)E(t+\tau)\rangle}{\langle I(t)\rangle}$$

Spectrum of the signal:

$$F(\omega) = \frac{1}{2\pi}\int exp(i\omega\tau)g^{(1)}(\tau)d\tau$$

Basis of Fourier Transform Spectroscopy

Hanbury Brown and Twiss Intensity-Intensity Correlations



$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$

Cauchy-Schwarz

$$2I(t)I(t+\tau) \leq I^2(t) + I^2(t+\tau)$$

The correlation is largest at equal time

$$g^{(2)}(0) \geq 1$$

$$g^{(2)}(0) \geq g^{(2)}(\tau)$$

Intensity correlation function measurements:

$$g^{(2)}(\tau) = \frac{\left\langle :\hat{I}(t)\hat{I}(t+\tau): \right\rangle}{\left\langle \hat{I}(t) \right\rangle^2}$$

Gives the probability of detecting a photon at time $t + \tau$ given that one was detected at time t. This is a conditional measurement:

$$g^{(2)}(\tau) = \frac{\left\langle \hat{I}(\tau) \right\rangle_c}{\left\langle \hat{I} \right\rangle}$$

Correlation function; Conditional measurement.

Detect a photon: get a conditional state.

The system has to have at least two photons.

Do we have enough signal to noise ratio?

$|LO|^2$ + $2\ LO\ S \cos(\phi)$

SHOT NOISE SIGNAL

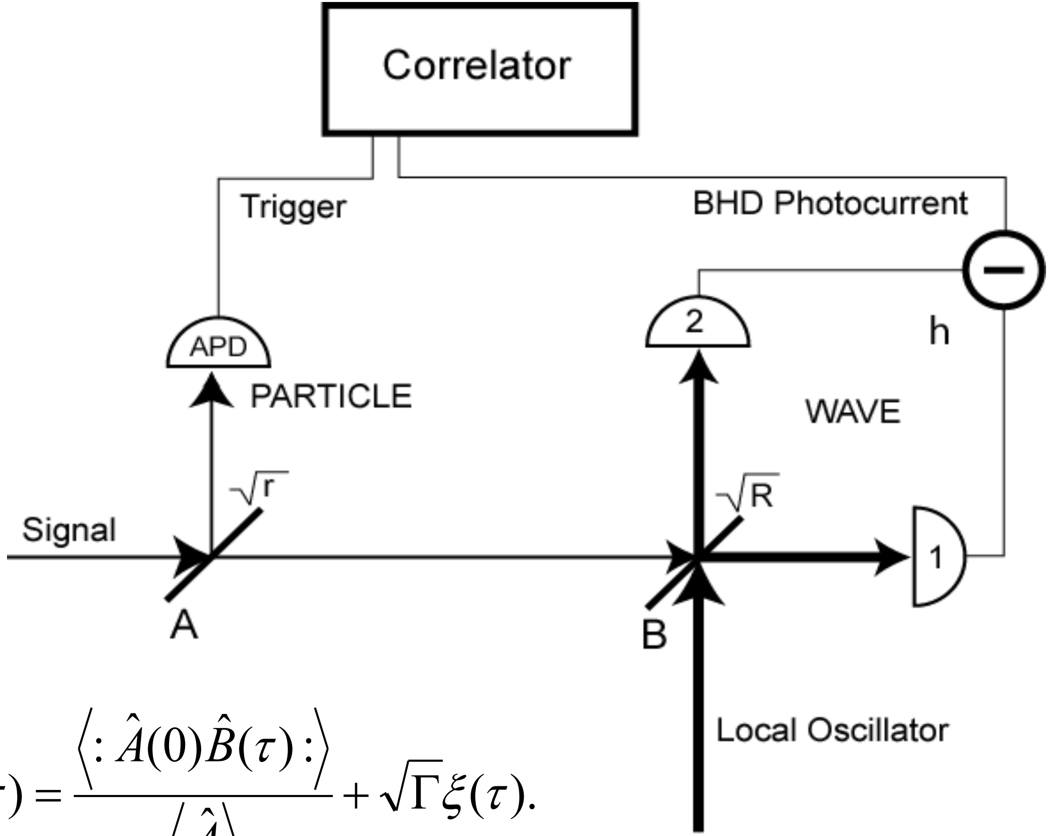
How to correlate fields and intensities?

Detection of the field: Homodyne.

Conditional Measurement: Only measure when we get a photon click.

Source: Cavity QED

The Intensity-Field correlator.



$$\mathrm{H}(\tau) = \frac{\left\langle : \hat{A}(0)\hat{B}(\tau) : \right\rangle}{\left\langle \hat{A} \right\rangle} + \sqrt{\Gamma}\,\xi(\tau).$$

Condition on a Click

Measure the correlation function of the Intensity and the Field:

$\langle I(t)\, E(t+\tau)\rangle$

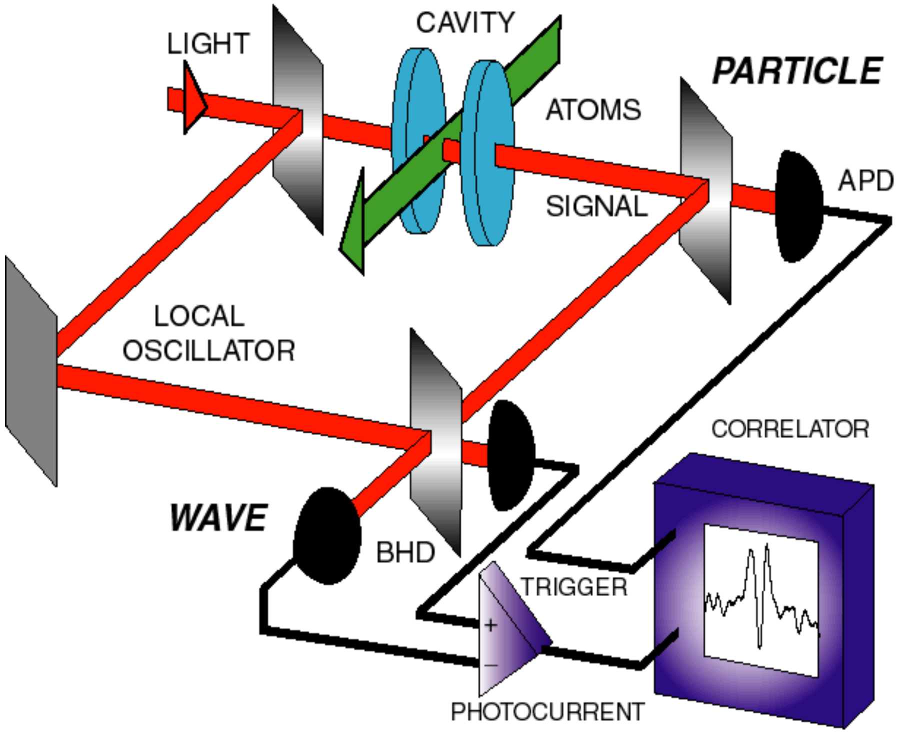
Normalized form:

$g^{(3/2)}{}_{\theta}(\tau) = h_{\theta}(\tau) = \langle E(\tau)\rangle_c / \langle E\rangle$

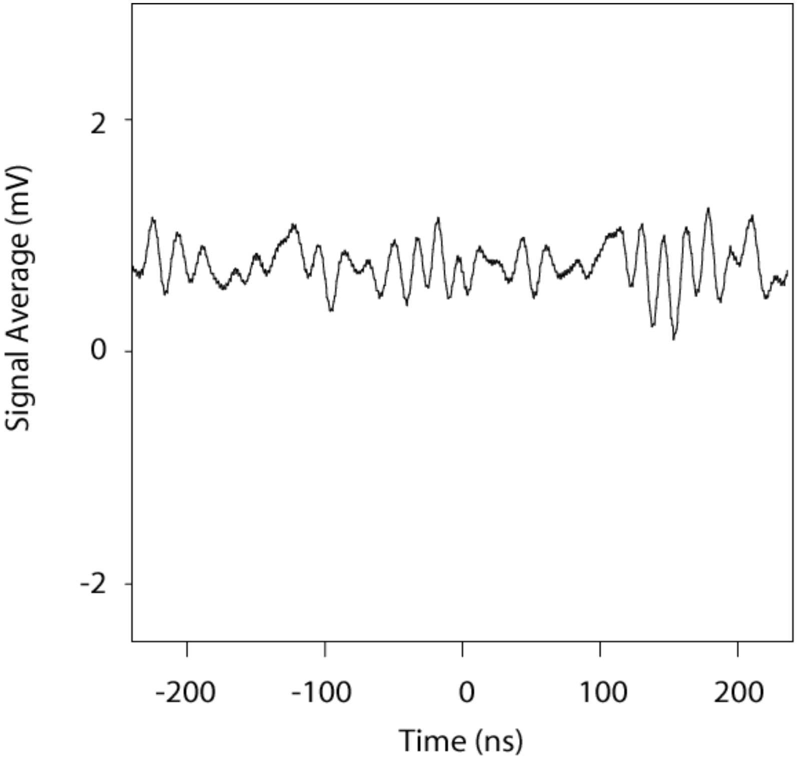
From Cauchy Schwartz inequalities:

$$0 \le \bar{h}_0(0) - 1 \le 2$$

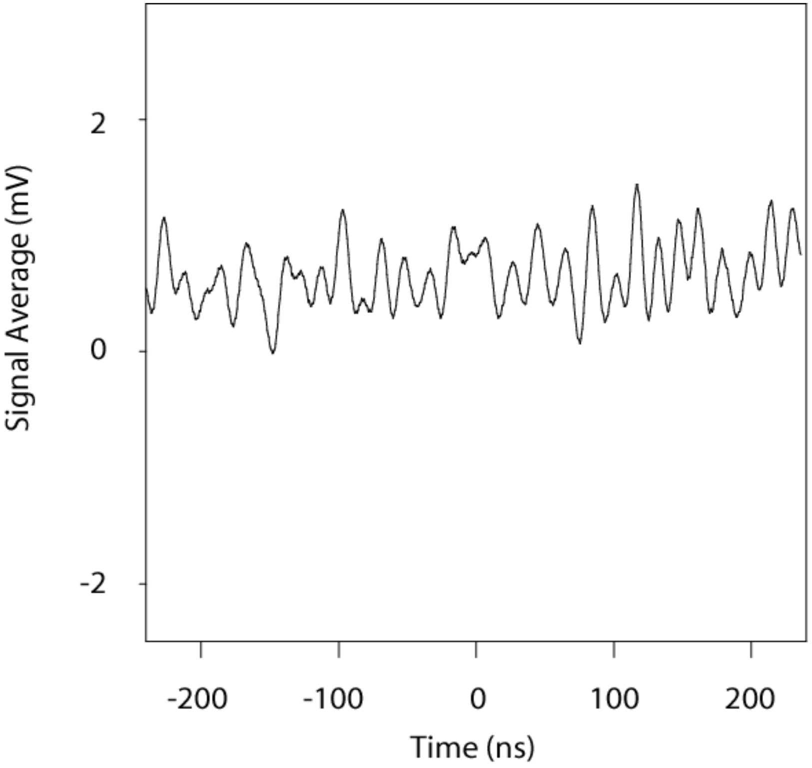
$$\left|\bar{h}_0(\tau) - 1\right| \le \left|\bar{h}_0(0) - 1\right|$$

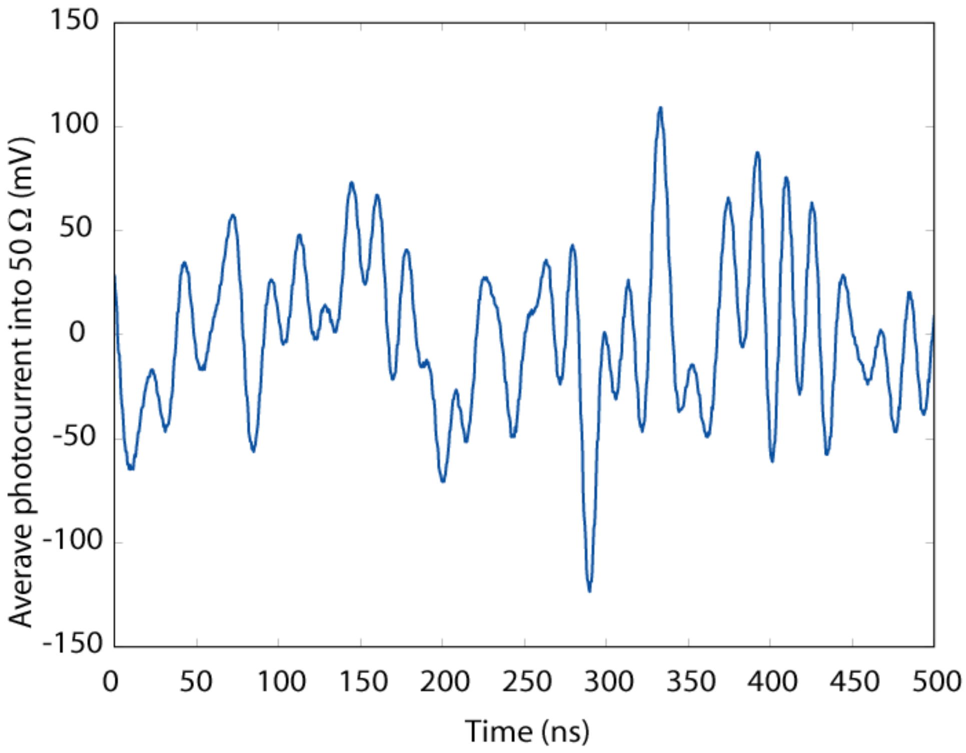

LIGHT
CAVITY
PARTICLE
ATOMS
APD
SIGNAL
LOCAL
OSCILLATOR
CORRELATOR
WAVE
BHD
TRIGGER
+
–
PHOTOCURRENT

Photocurrent average with random conditioning

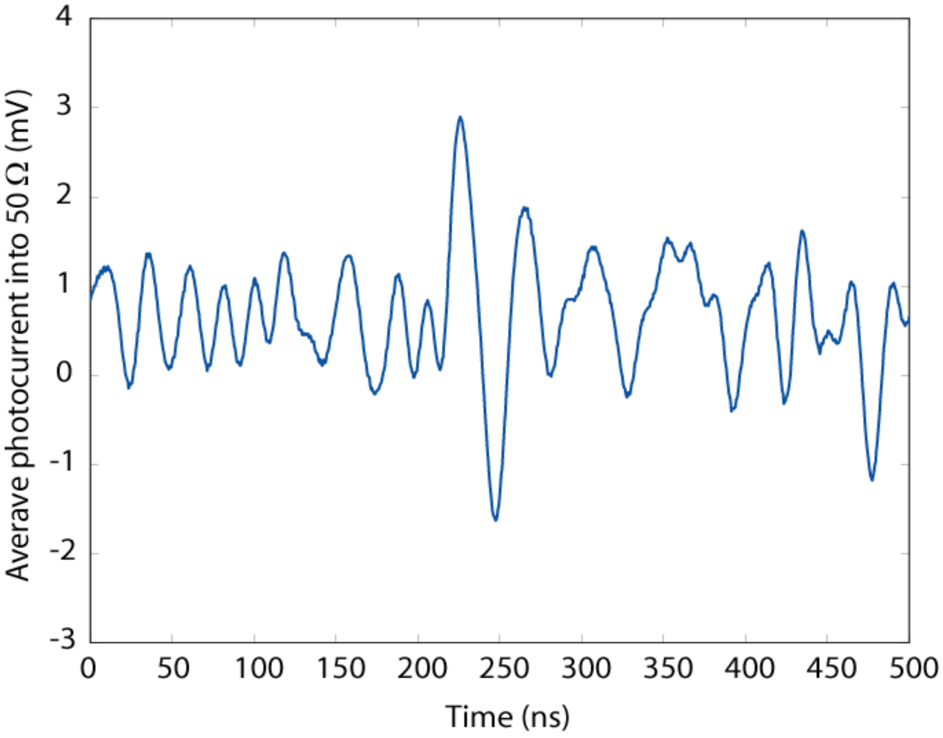


Conditional photocurrent with no atoms in the cavity.

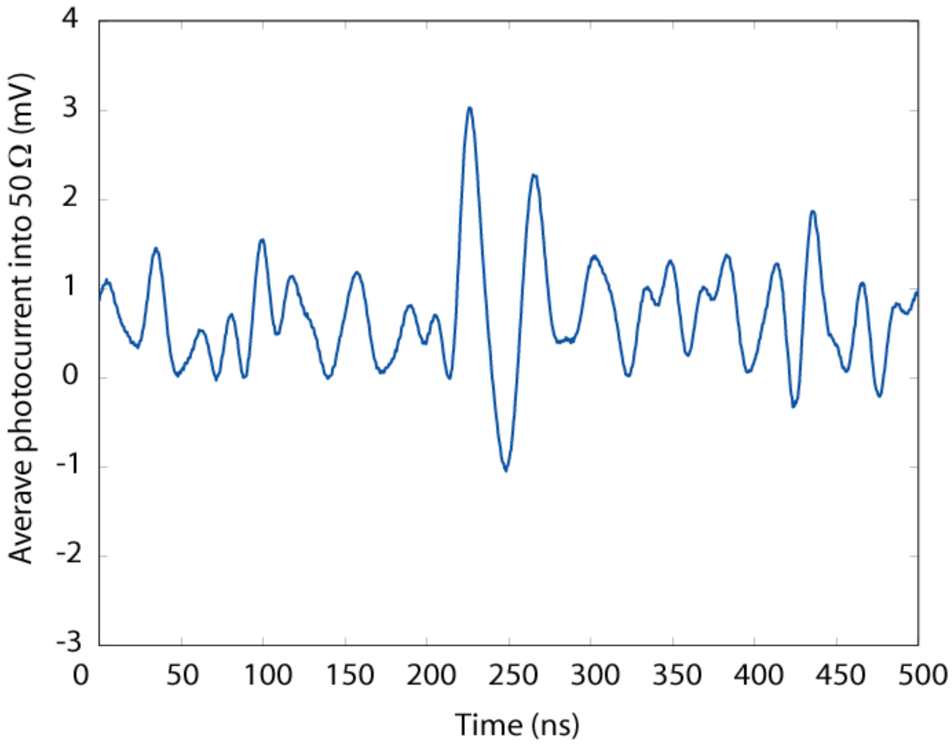




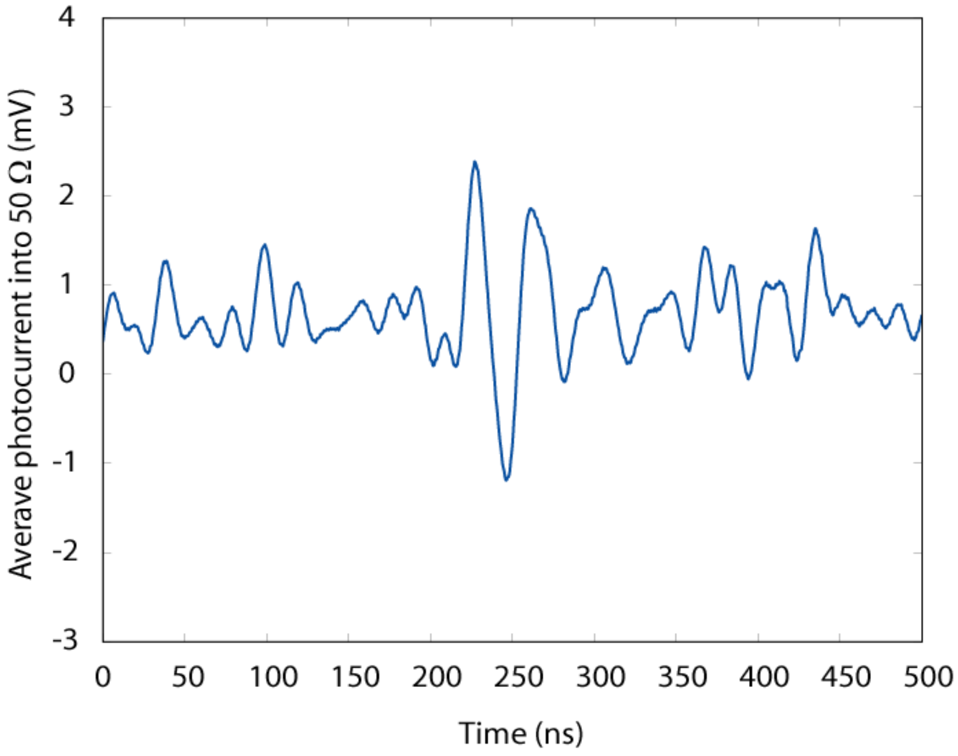
After 1 average, pp~200 mV



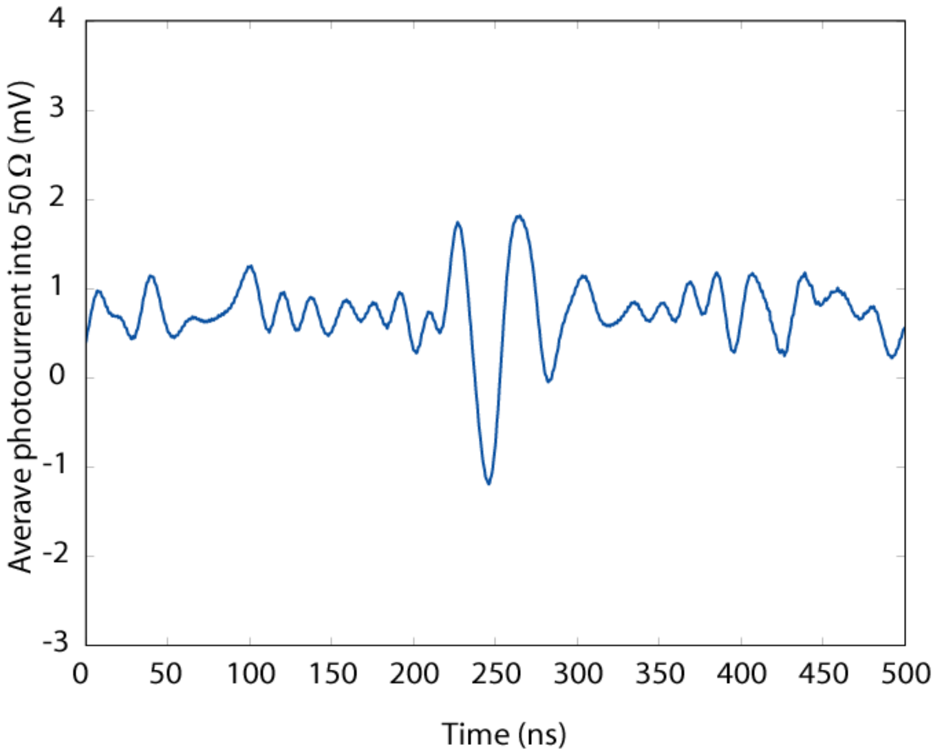
After 6,000 averages



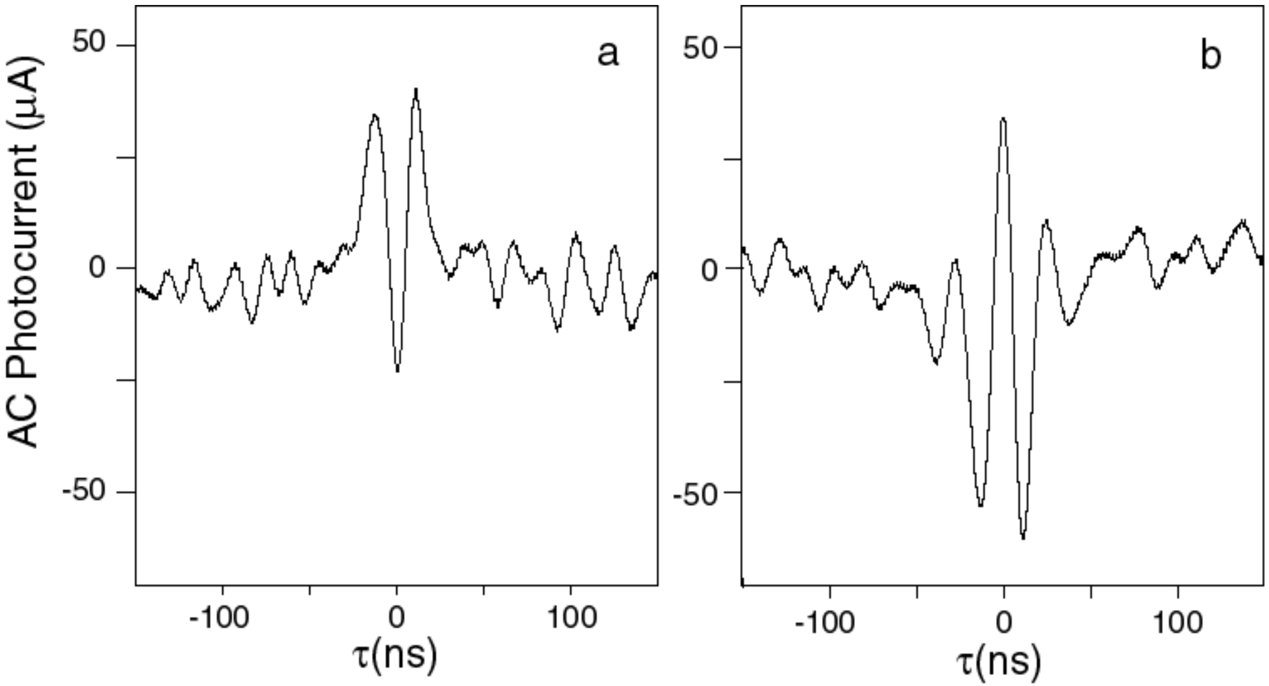
After 10,000 averages



After 30,000 averages

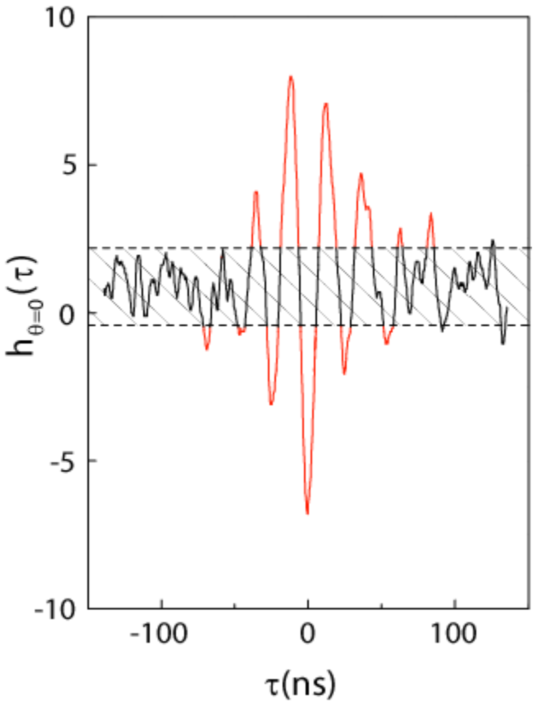


After 65,000 averages, noise pp~1mV



Flip the phase of the Mach-Zehnder by 146°

Monte Carlo simulations for weak excitation:



in black the classically allowed region

Atomic beam N=11

This is the conditional evolution of the field of a fraction of a photon [B(t)] from the correlation function.

$h_{\theta}(\tau) = \langle E(\tau)\rangle_c / \langle E\rangle$

The conditional field prepared by the click is:

$A(t)|0\rangle + B(t)|1\rangle$ with $A(t) \approx 1$ and $B(t) << 1$

We measure the field of a fraction of a photon!

Fluctuations are very important.

Conditional evolution of the state one atom

$$|\Psi_{ss}\rangle = |0,g\rangle + \lambda|1,g\rangle - \frac{2g}{\gamma}\lambda|0,e\rangle + \frac{\lambda^2 pq}{\sqrt{2}}|2,g\rangle - \frac{2g\lambda^2 q}{\gamma}|1,e\rangle$$

$$\lambda = \langle\hat{a}\rangle,\quad p = p(g,\kappa,\gamma) \text{ and } q = q(g,\kappa,\gamma) \qquad \lambda = \frac{\varepsilon}{\kappa}\left(\frac{1}{1+2C}\right)$$

$$p = 1 - 2C_1', \quad q = (1+2C)/(1+2C-2C_1') \quad \text{with } C_1' = C_1(1+\gamma/2\kappa)^{-1}$$

$$\hat{a}|\Psi_{ss}\rangle \Rightarrow |\Psi_{conditioned}\rangle = |0,g\rangle + \lambda pq|1,g\rangle - \frac{2g\lambda q}{\gamma}|0,e\rangle$$

Field atomic polarization

$$g^{(3/2)}(0) = |pq| = 1 - \frac{4C_1^2}{(1+\gamma/2\kappa)(1-2C_1)-2C_1}$$

For N non-interacting atoms

$$|\chi(t)\rangle = |00\rangle + A_1(t)|10\rangle + A_2(t)|01\rangle$$
$$+ A_3(t)|20\rangle + A_4(t)|11\rangle + A_5(t)|02\rangle$$

Equations of motion of the coefficients

$$\dot{A}_1 = -\kappa A_1 + \sqrt{N} g A_2 + \mathcal{E}$$
$$\dot{A}_2 = -(\gamma/2) A_2 - \sqrt{N} g A_1$$
$$\dot{A}_3 = -2\kappa A_3 + \sqrt{2}\sqrt{N} g A_4 + \sqrt{2}\mathcal{E} A_1 ,$$
$$\dot{A}_4 = -(\kappa + \gamma/2) A_4 - \sqrt{2}\sqrt{N} g A_3 + \sqrt{2}\sqrt{N-1} g A_5 + \mathcal{E} A_2 ,$$
$$\dot{A}_5 = -\gamma A_5 - \sqrt{2}\sqrt{N-1} g A_4 .$$

Field with drive $\mathcal{E}$

Polarization

$$g^{(3/2)}(\tau) = 1 + \mathcal{A}\mathcal{F} \tag{17}$$

where $\mathcal{F}$ is a decaying oscillation,

$$\mathcal{F} = e^{-\beta\tau}[\cos(\Omega_0\tau) + (\beta/\Omega_0)\sin(\Omega_0\tau)], \tag{18}$$

with $\beta \equiv (\kappa + \gamma_\perp)/2$ representing the average decay rate and Ω_0 the vacuum Rabi frequency in the low intensity limit:

$$\Omega_0 = \sqrt{g^2 N - \frac{(\kappa - \gamma_\perp)^2}{4}}.$$

The amplitude of the decaying oscillations is given by

$$\mathcal{A} = -\frac{4C_1^2 N}{(1+\gamma_\perp/\kappa)(1+2C_1 N) - 2C_1}.$$

This is the conditional evolution of the field of a fraction of a photon [B(t)] from the correlation function.

$$h_\theta(\tau) = \langle E(\tau)\rangle_c / \langle E\rangle$$

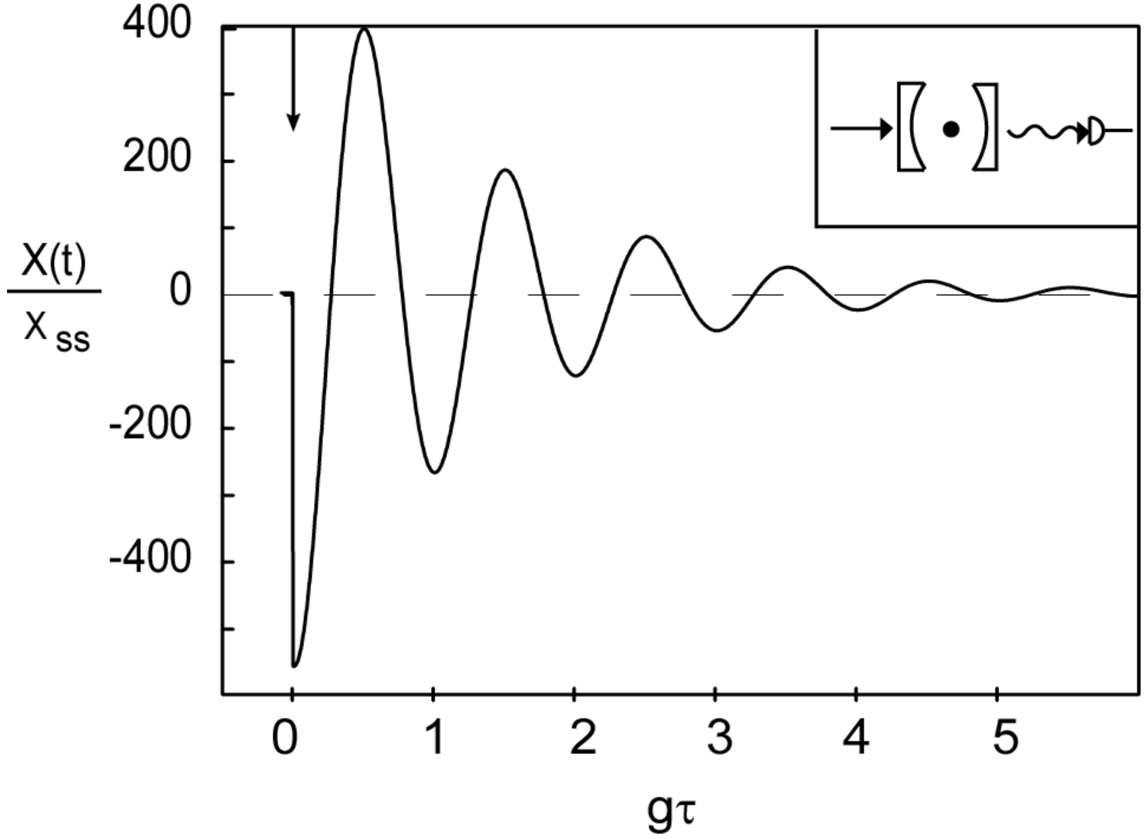
The conditional field prepared by the click is:

$A(t)|0\rangle + B(t)|1\rangle$ with $A(t) \approx 1$ and $B(t) \ll 1$

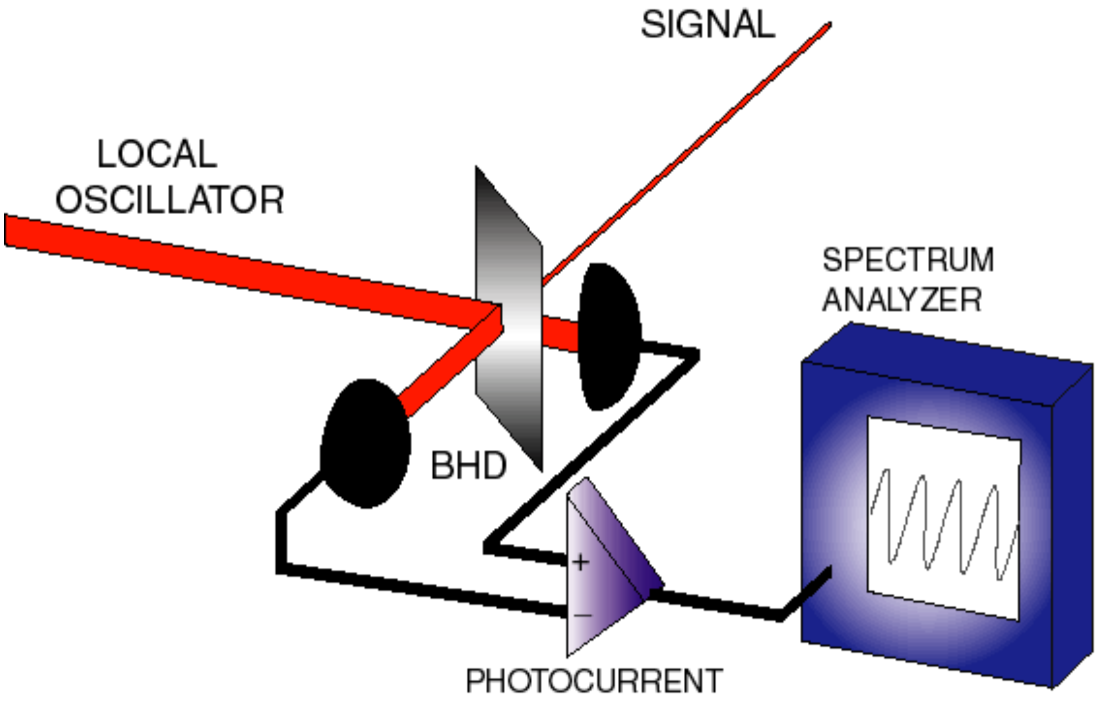
We measure the field of a fraction of a photon!

Fluctuations are very important.

Regression of the field to steady state after the detection of a photon.



Detection of the Squeezing spectrum with a balanced homodyne detector (BHD).

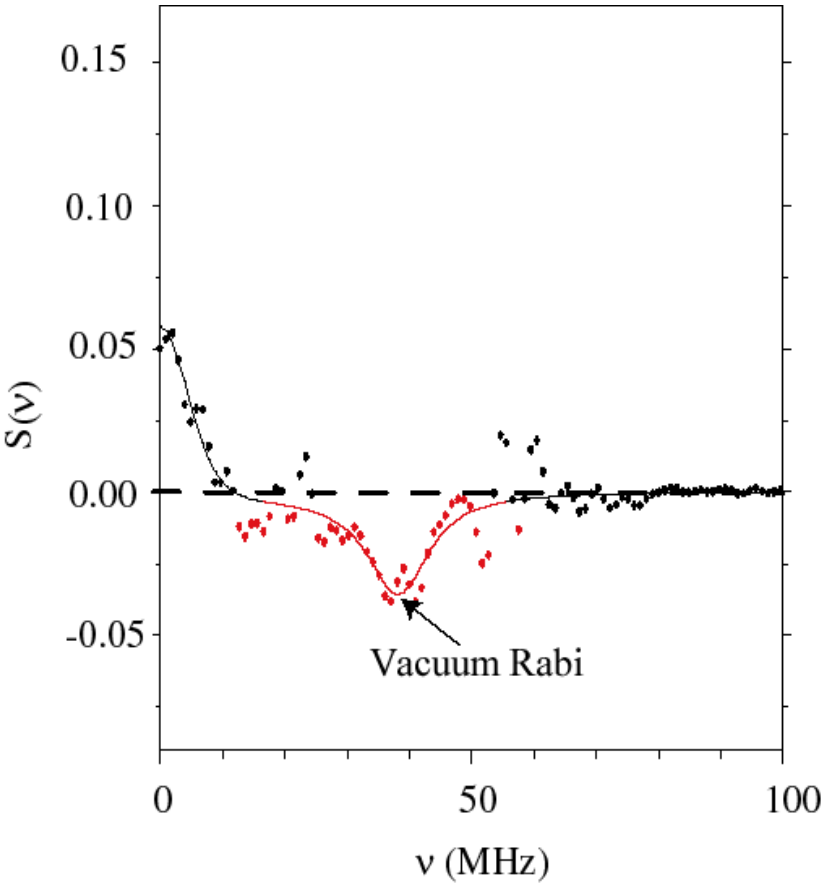


The fluctuations of the electromagnetic field are measured by the spectrum of squeezing. Look at the noise spectrum of the photocurrent.

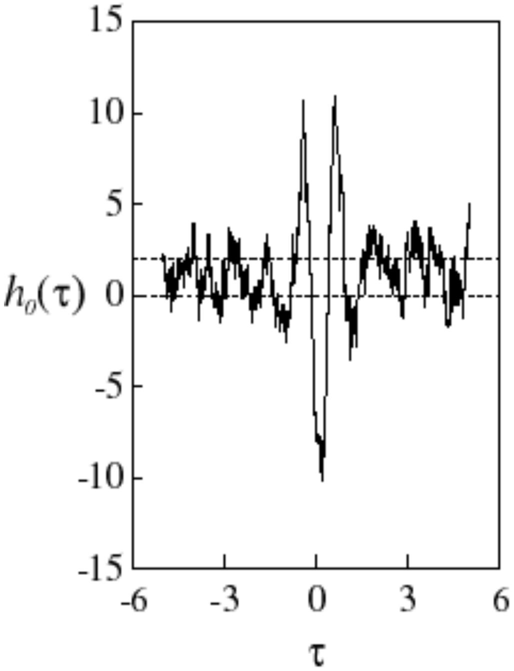
$$S(\nu, 0^{\circ}) = 4F\int_{0}^{\infty} \cos(2\pi\nu\tau)[\bar{h}_0(\tau) - 1]d\tau,$$

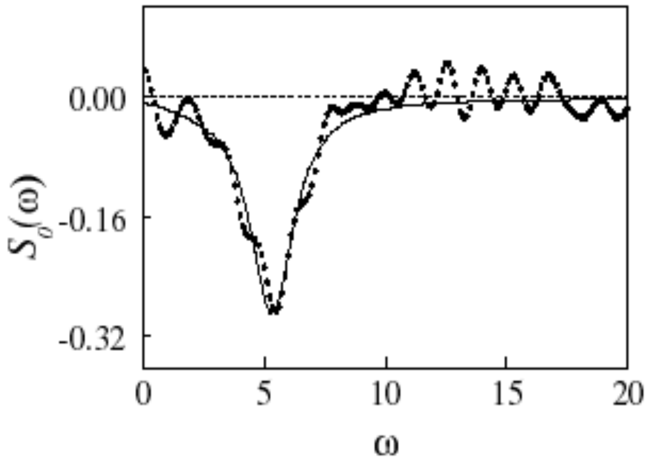
F is the photon flux into the correlator.

Spectrum of Squeezing from the F. T. of $g^{(3/2)}(\tau)=h_0(\tau)$

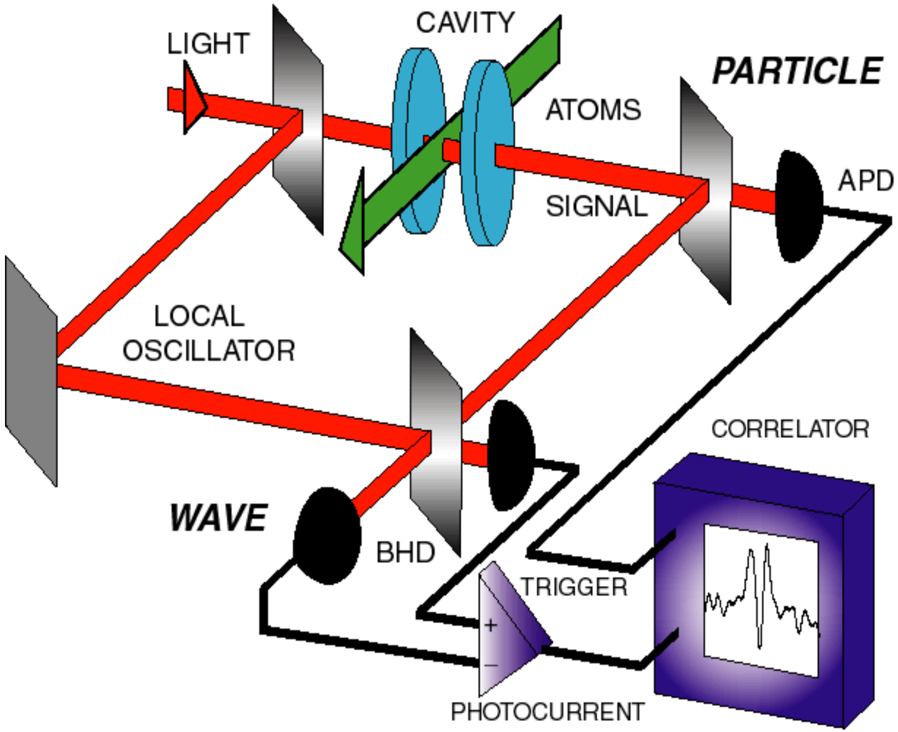


Monte Carlo simulations of the wave-particle correlation and the spectrum of squeezing in the low intensity limit for an atomic beam.





It has upper and a lower classical bounds


CAVITY
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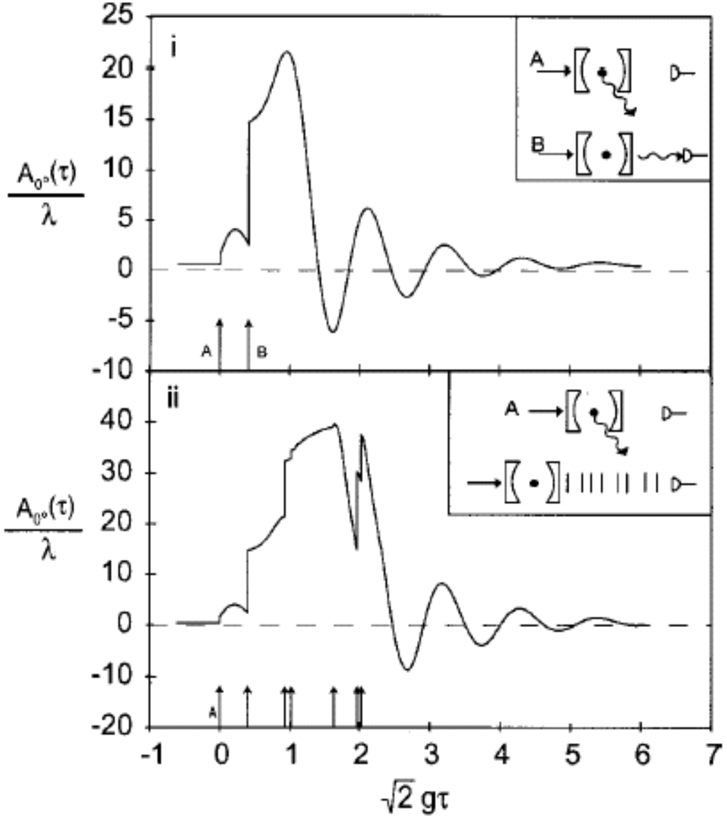
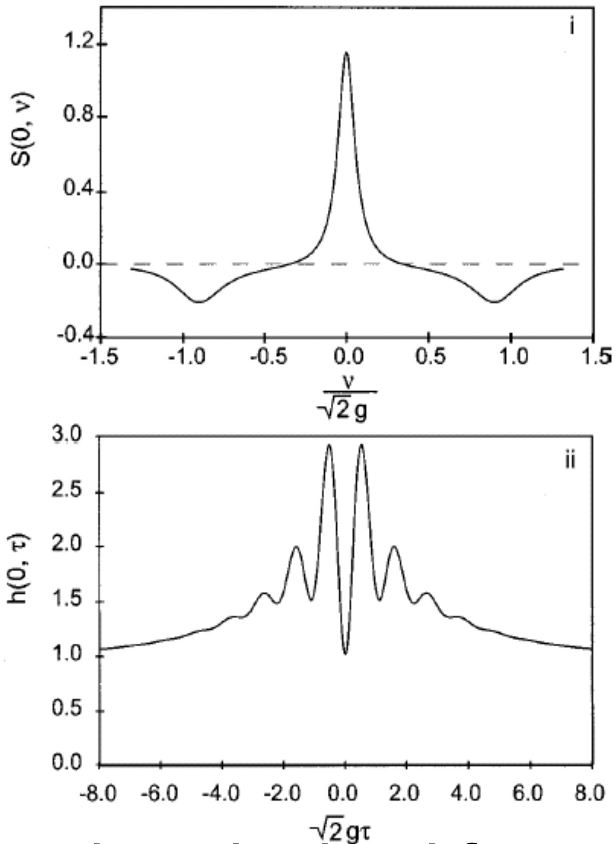
PHOTOCURRENT

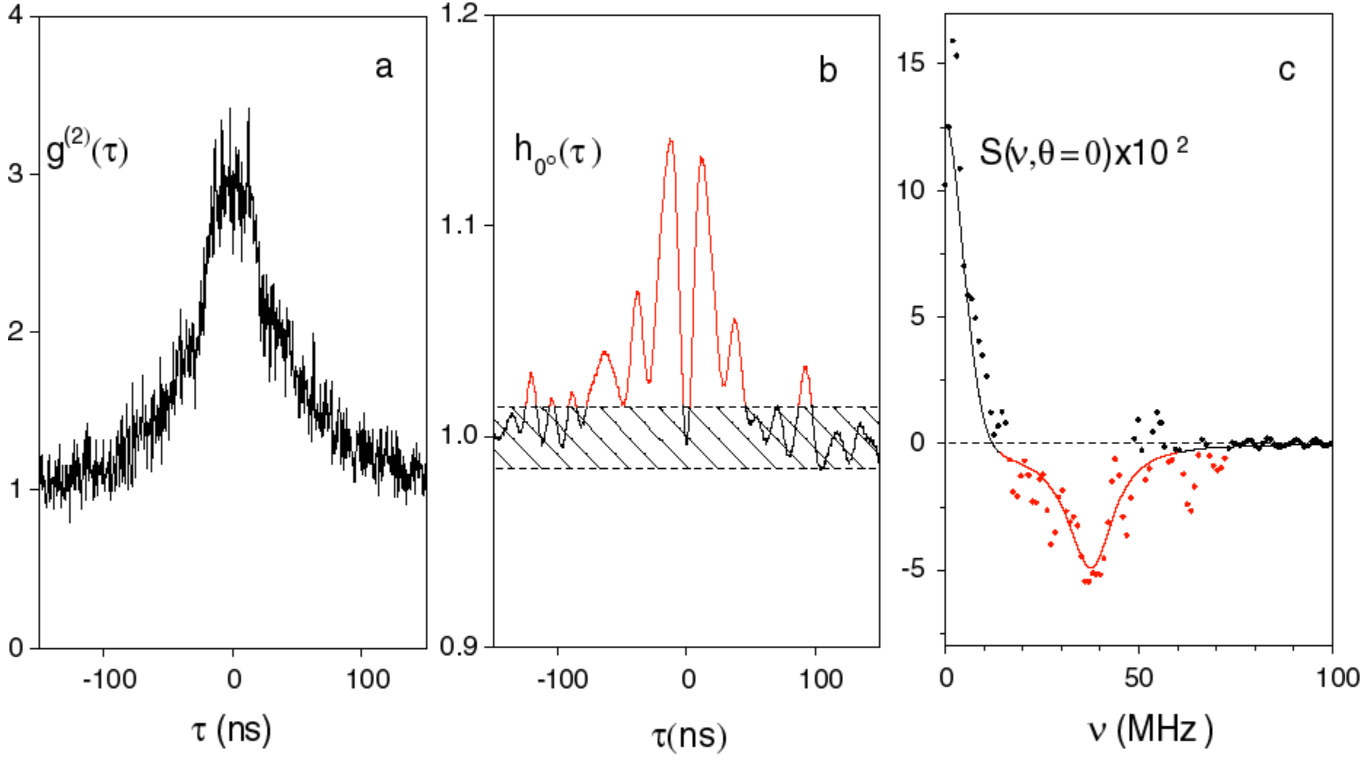


Single quantum trajectories simulation of cavity QED system with spontaneous emission.



(i) Spectrum of squeezing obtained from the averaged (ii) $h_0(\tau)$ correlation function that shows the effects of spontaneous emission.

Classical $g^{(2)}$ | Non-classical h | Squeezing



N=13; $1.2n_0$

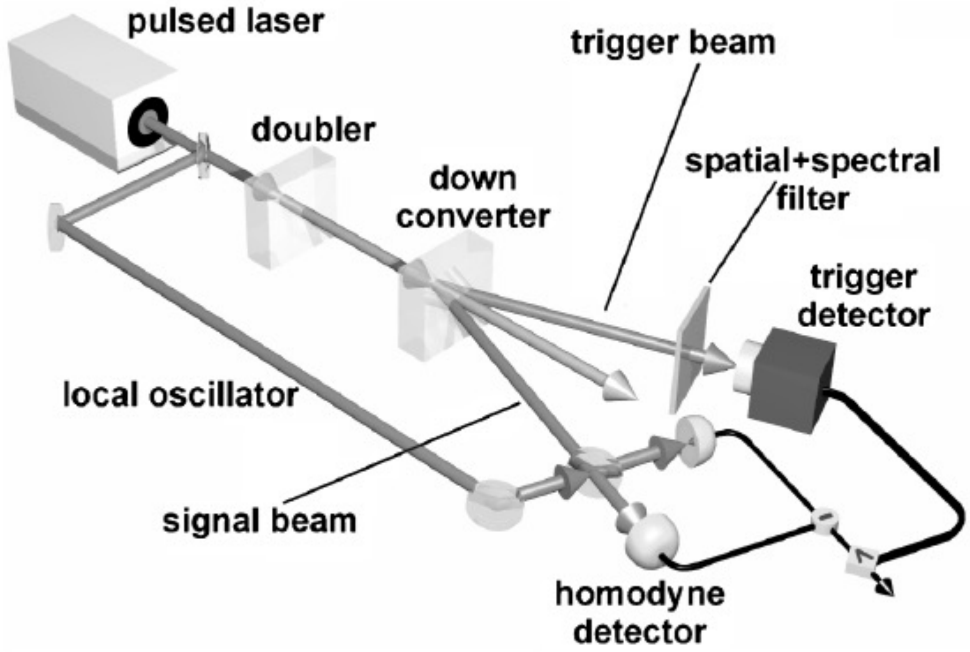
Other systems

Twin photons from down conversion

Quantum State Reconstruction of the Single-Photon Fock State

A. I. Lvovsky,* H. Hansen, T. Aichele, O. Benson, J. Mlynek,† and S. Schiller‡

Phys. Rev. Lett. 87, 050402 (2001)



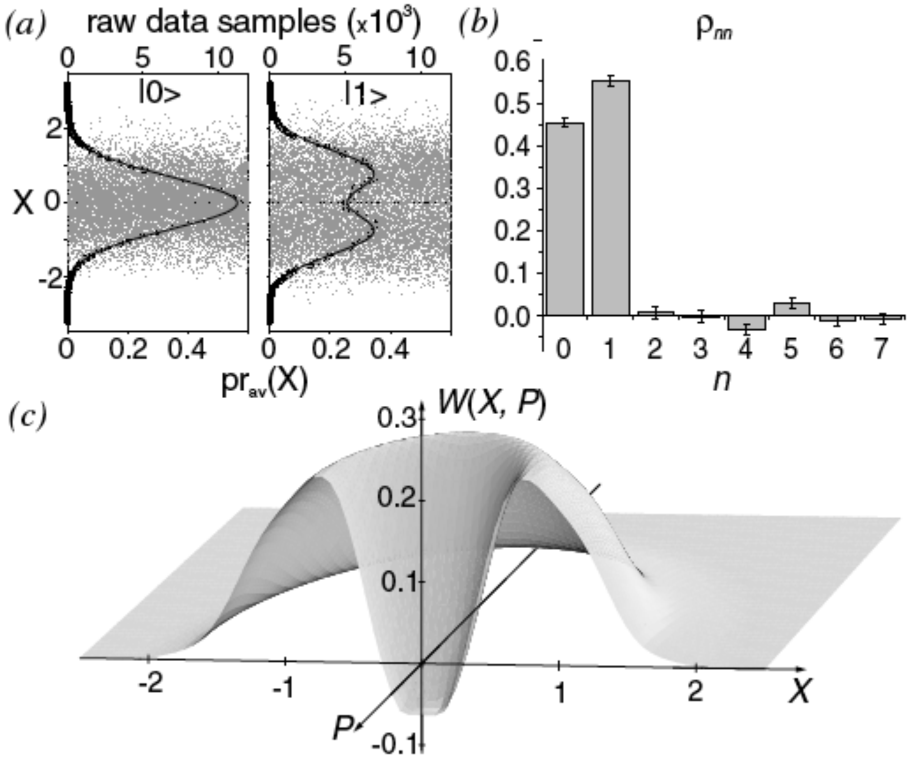
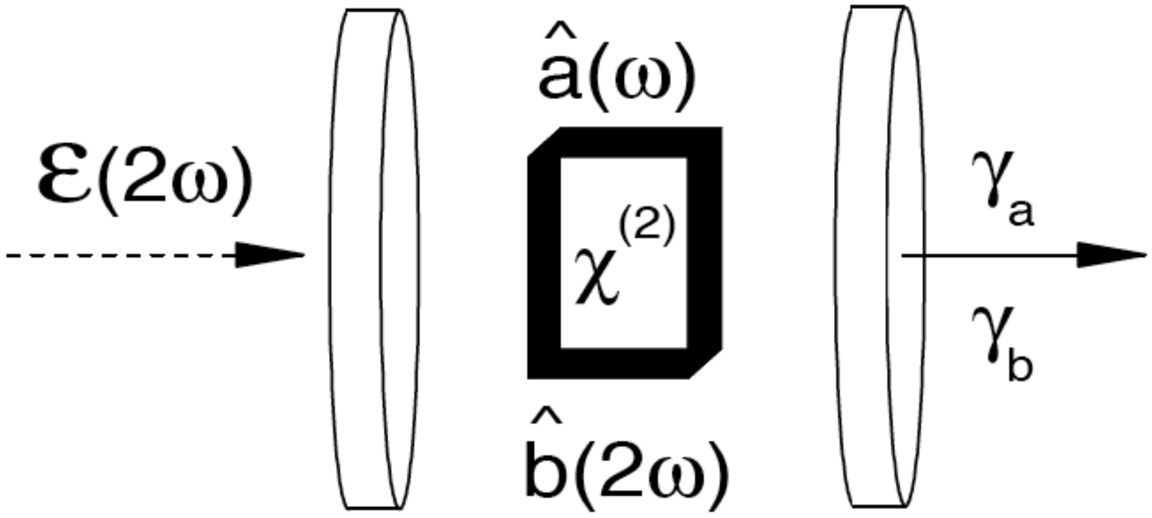


FIG. 4. Experimental results: (a) raw quantum noise data for the vacuum (left) and Fock (right) states along with their histograms corresponding to the phase-randomized marginal distributions; (b) diagonal elements of the density matrix of the state measured; (c) reconstructed WF which is negative near the origin point. The measurement efficiency is 55%.

Optical Parametric Oscillator



$$H = \hbar\omega_a \hat{a}^\dagger \hat{a} + \hbar\omega_b \hat{b}^\dagger \hat{b} + \frac{i\hbar\chi}{2}(\hat{a}^{\dagger 2}\hat{b} - \hat{a}^2\hat{b}^\dagger).$$

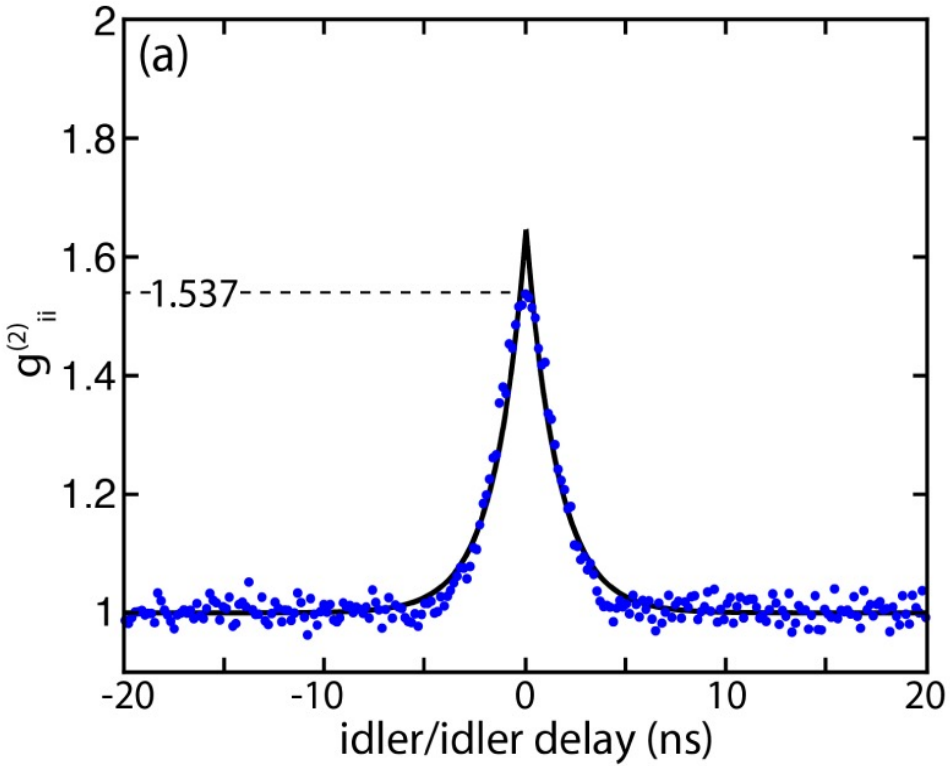
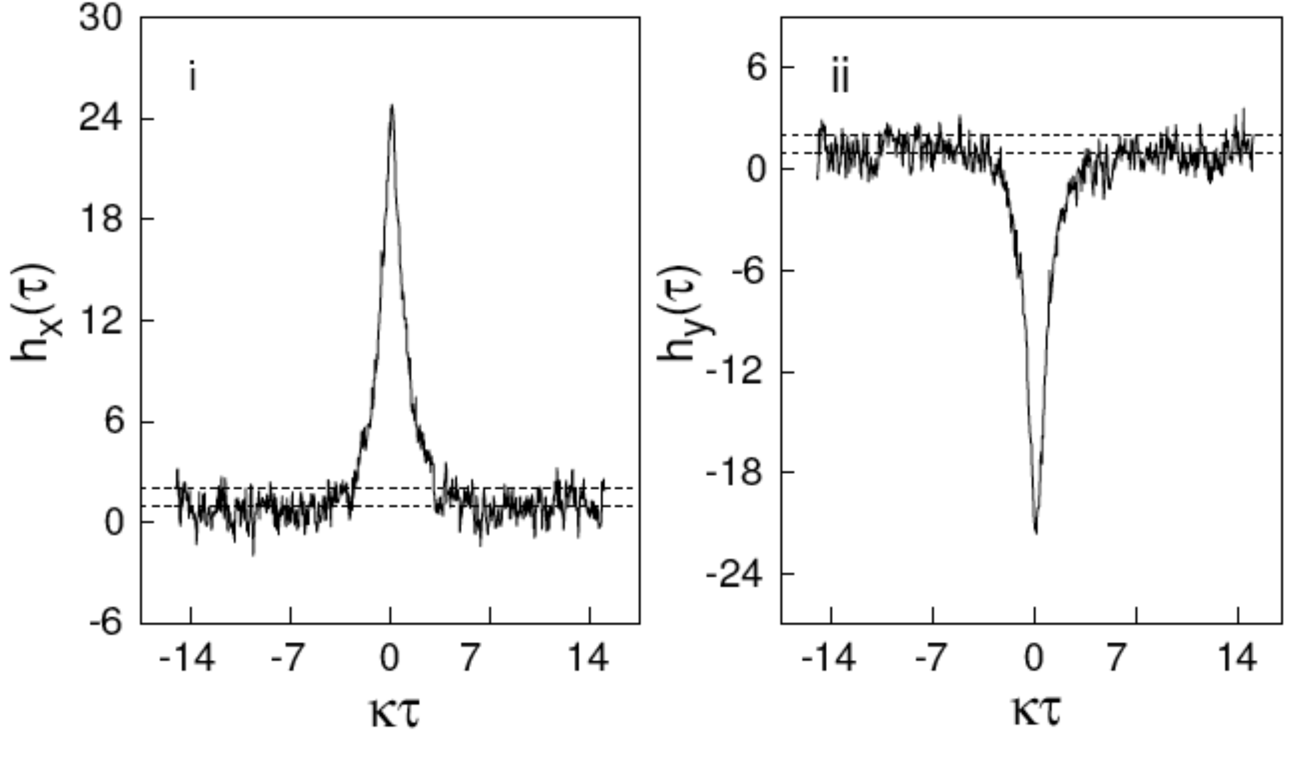


Fig. 4

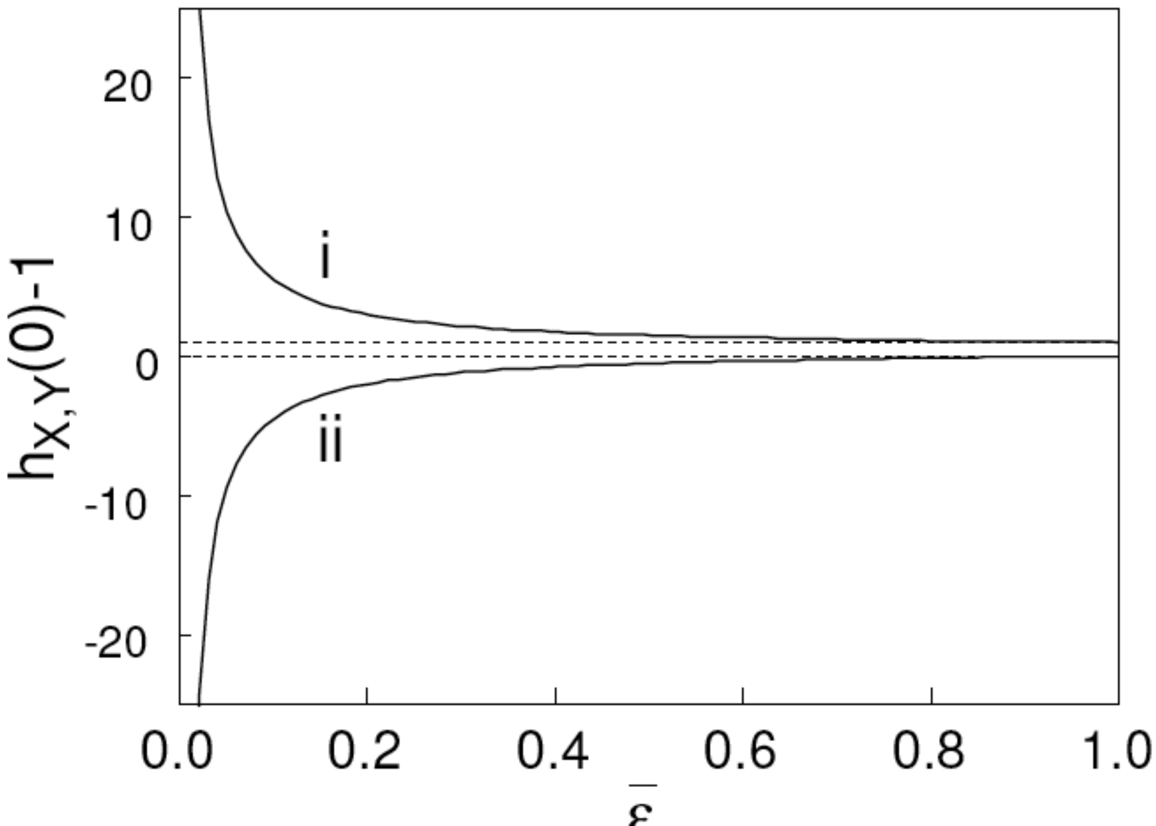
Citation

Christian Reimer, Lucia Caspani, Matteo Clerici, Marcello Ferrera, Michael Kues, Marco Peccianti, Alessia Pasquazi, Luca Razzari, Brent E. Little, Sai T. Chu, David J. Moss, Roberto Morandotti, "Integrated frequency comb source of heralded single photons," Opt. Express **22,** 6535-6546 (2014);
https://www.osapublishing.org/oe/abstract.cfm?uri=oe-22-6-6535

Calculation of $h_\theta(\tau)$ in an OPO well below threshold with the classical bounds



Maximum of $h_\theta(\tau)$ in an OPO below threshold



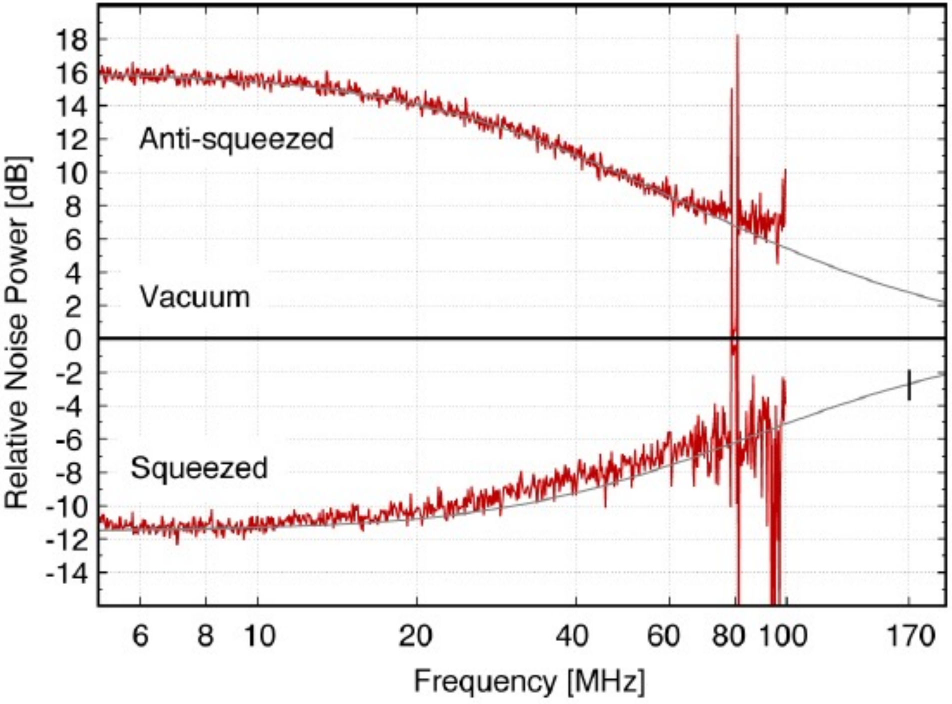


FIG. 6: High bandwidth squeezing spectrum. Squeezing (bottom trace) and anti-squeezing (top trace) are shown relative to the vacuum noise variance. The measurements were performed from 5 MHz

M. Mehmet, H. Vahlbruch, N. Lastzka, K. Danzmann, and R. Schnabel, Phys. Rev. A **81**, 013814.

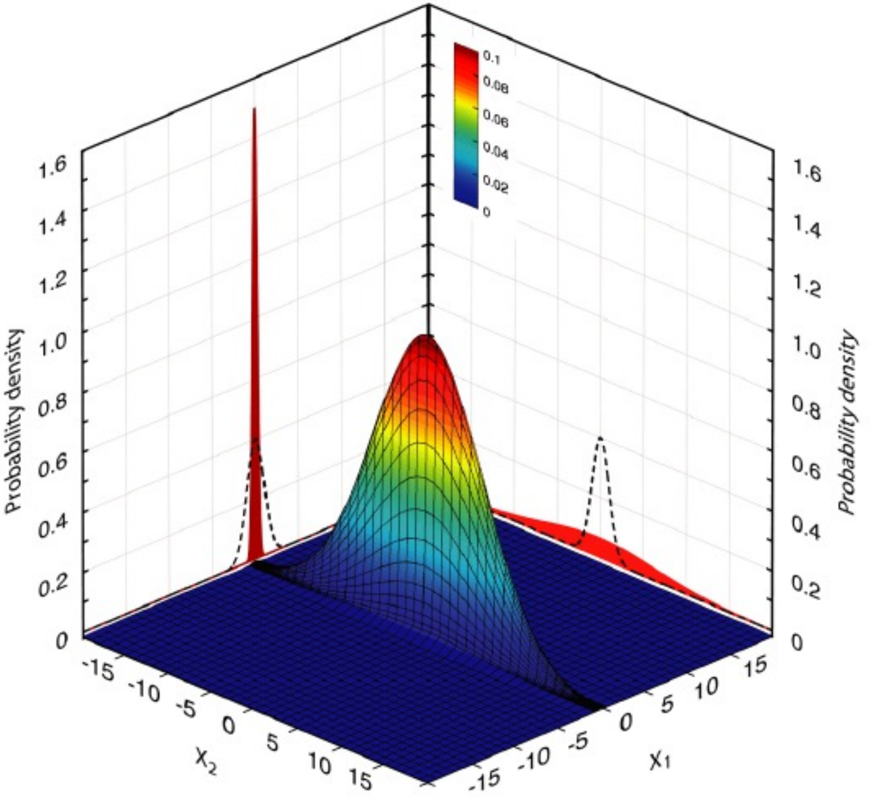


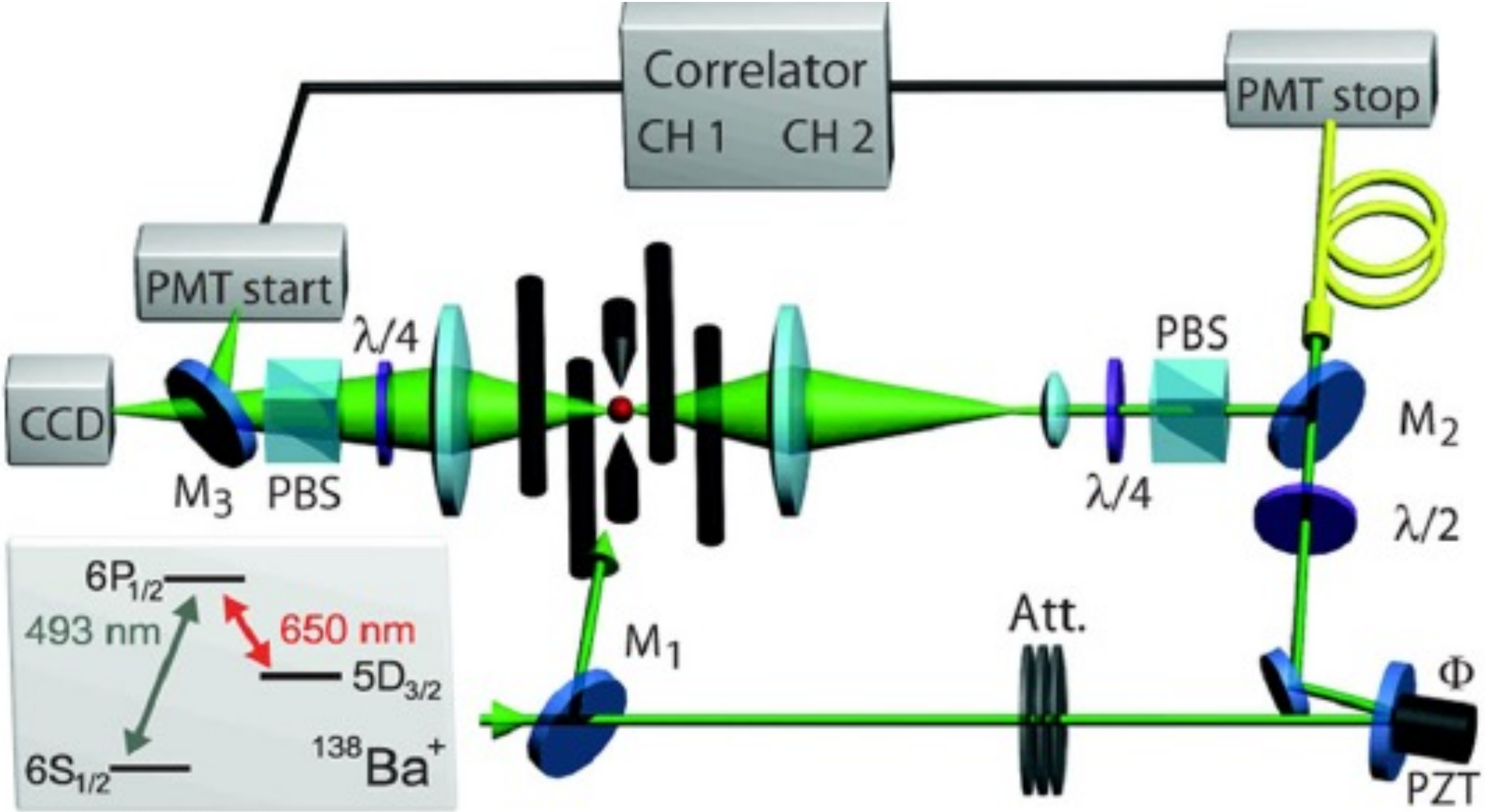
FIG. 3: Wigner function of the squeezed vacuum state produced by our OPO. The projections (filled curves) onto the two quadratures yield the gaussian probability distributions with variances of -11.5 dB and 16 dB relative to the projections belonging to a pure vacuum state (dotted curves).

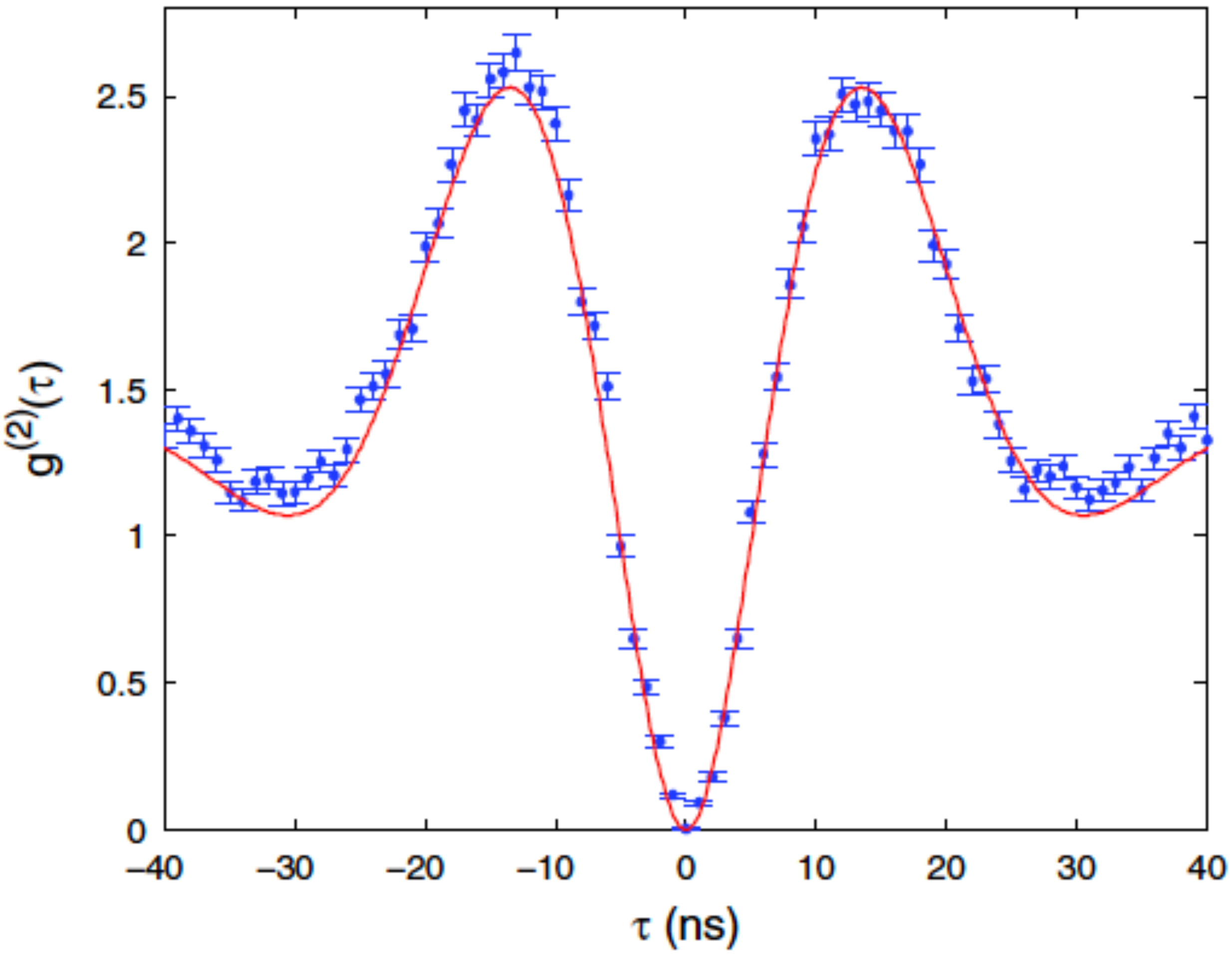
Single ion resonance fluorescence

PRL **102,** 183601 (2009) PHYSICAL REVIEW LETTERS week ending 8 MAY 2009

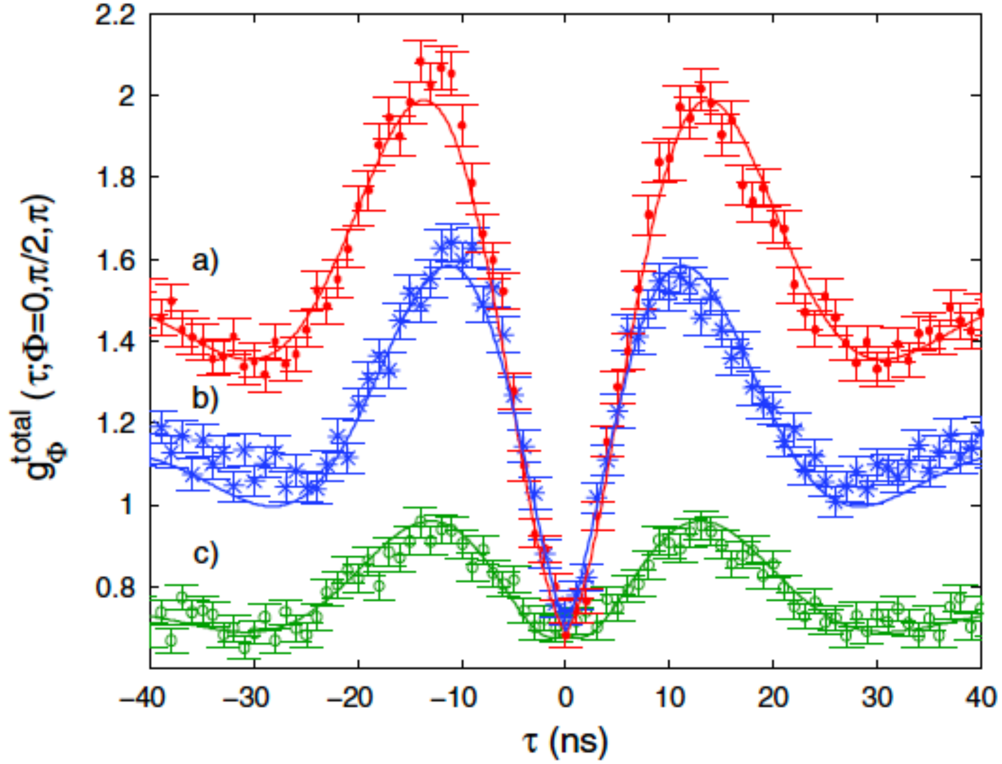
Intensity-Field Correlation of Single-Atom Resonance Fluorescence

S. Gerber,[1] D. Rotter,[1] L. Slodička,[1] J. Eschner,[1,4] H. J. Carmichael,[3] and R. Blatt[1,2]

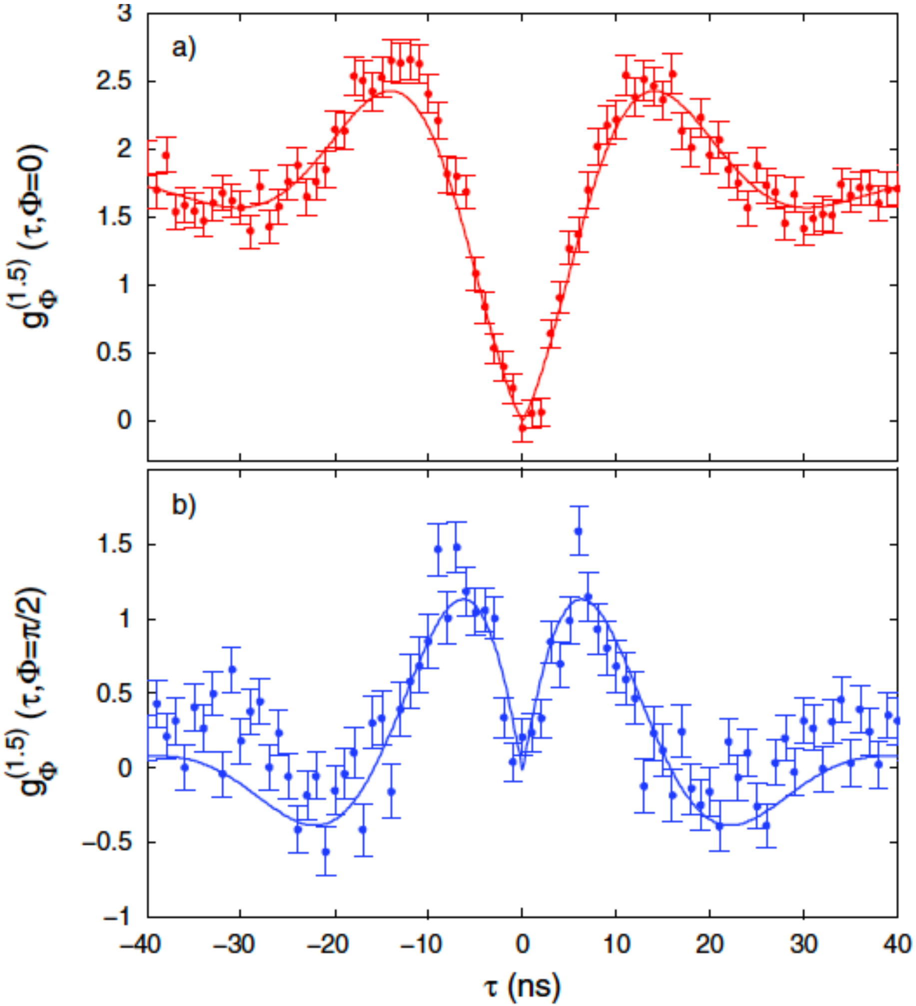

Correlator
CH 1
CH 2
PMT stop
PMT start
λ/4
PBS
CCD
M3
PBS
M2
λ/4
λ/2
6P1/2
493 nm
650 nm
5D3/2
6S1/2
138Ba+
M1
Att.
Φ
PZT



2.5
2
1.5
1
0.5
0
−40
−30
−20
−10
0
10
20
30
40
g(2)(τ)
τ (ns)



$$g_{\Phi}^{\text{total}}(\tau) = (1 - r) + rg^{(2)}(\tau) + \frac{V}{1 - V} g_{\Phi}^{(1.5)}(\tau).$$



a)
b)
$g_\Phi^{(1.5)}(\tau,\Phi=0)$
$g_\Phi^{(1.5)}(\tau,\Phi=\pi/2)$
τ (ns)

- The wave-particle correlation $h_\theta(\tau)=g^{(3/2)}{}_\theta(\tau)$ measures the conditional dynamics of the electromagnetic field. The Spectrum of Squeezing $S(\Omega)$ and $h_\theta(\tau)$ are Fourier Transforms of each other.
- Many applications in many other problems of quantum optics and of optics in general: microscopy, degaussification, weak measurements, quantum feedback.
- Possibility of a tomographic reconstruction of the dynamical evolution of the electromagnetic field state.

Bibliography

H. J. Carmichael, G. T. Foster, L. A. Orozco, J. E. Reiner, and P. R. Rice, “Intensity-Field Correlations of Non-Classical Light”. Progress in Optics, Vol. 46, 355-403, Edited by E. Wolf Elsevier, Amsterdam 2004.

Thanks